Given a positive integer n, n factorial (written $n!$) is a shorthand for the product of all positive integers less than or equal to the given integer n.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. (By convention, we say that $0! = 1$.)

1. Evaluate the following.

   (a) $5!$ $\quad 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

   (b) $6!$ $\quad 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

   (c) $\dfrac{6!}{5!}$ $\quad \dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 6$

   (d) $\dfrac{5!}{6!}$ $\quad \dfrac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{1}{6}$

   (e) $\dfrac{102!}{100!}$ $\quad \dfrac{102 \cdot 101 \cdot 100!}{100!} = 102 \cdot 101$

2. Simplify each of the following. Assume n is a positive integer.

   (a) $\dfrac{(n+2)!}{n!}$ $\quad (n+2)(n+1)$

   (b) $\dfrac{(n-3)!}{n!}$ $\quad \dfrac{1}{n(n-1)(n-2)}$

   (c) $\dfrac{(2n+2)!}{(2n)!}$ $\quad (2n+2)(2n+1)$

   (d) $\dfrac{(2n+2)!}{2n!}$ $\quad \dfrac{(2n+2)(2n+1)(2n)\cdots(n+1)}{2}$

   (e) $\dfrac{(n!)^2}{((n+1)!)^2}$ $\quad \dfrac{n! \cdot n!}{(n+1)! \cdot (n+1)!} = \dfrac{1}{n+1} \cdot \dfrac{1}{n+1} = \dfrac{1}{(n+1)^2}$